

Strange-quark mass from combined e^+e^- and τ -decay data: test of the isospin symmetry and implications on ϵ'/ϵ and $m_{u,d}$

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Abstract

I extract the strange-quark mass using a τ -like decay sum rule for the ϕ -meson, and some other sum rules involving its difference with the vector component of the hadronic τ -decay. As a conservative estimate, one obtains to order α_s^3 : $\overline{m}_s(1 \text{ GeV}) = (178 \pm 33) \text{ MeV} \Rightarrow \overline{m}_s(2 \text{ GeV}) = (129 \pm 24) \text{ MeV}$, while the positivity of the spectral function leads to the upper bound: $\overline{m}_s(1 \text{ GeV}) \leq (200 \pm 28) \text{ MeV} \Rightarrow \overline{m}_s(2 \text{ GeV}) \leq (145 \pm 20) \text{ MeV}$. These results are in good agreement with the existing sum rule and τ -decay results, and, in particular, with the result from the the sum rule involving the difference of the isoscalar and isovector components of the $e^+e^- \rightarrow \text{hadrons}$ data. This signals small effects of the $SU(2)$ isospin violation due to the ω - ρ mixing parameters, and questions the reliability of the existing sum rule estimates of these parameters. Combining our result with the recent data on ϵ'/ϵ , we can estimate, within the standard model, the four-quark weak matrix elements $B_6^{1/2} - 0.54B_8^{3/2}$ to be about (2.8 ± 1.3) . This result may suggest a large violation of the vacuum saturation estimate similarly to the case of the four-quark condensates obtained from the sum rules analysis, and can serve as a guide for a future accurate non-perturbative extraction of such matrix elements. Combining our result with the sum rule estimate of $m_u + m_d$ and with the Dashen formula for the mass ratio, one can deduce the update values: $\overline{m}_d(2 \text{ GeV}) = (6.4 \pm 1.1) \text{ MeV}$ and $\overline{m}_u(2 \text{ GeV}) = (2.3 \pm 0.4) \text{ MeV}$.

1 Introduction

Among the most important parameters of the standard model and of chiral symmetry breaking are the light quark masses. In addition to a much better understanding of the realizations of chiral symmetry breaking [1], the recent measurement of the CP violating parameters ϵ'/ϵ [2] needs a much better control of the Standard Model predictions, which can be largely affected by the value of the running strange quark mass within the widely used parametrization of the non-perturbative four-quark matrix elements Q_6 and Q_8 [3]. A lot of effort reflected in the literature [4] has been put into extracting directly from the data the running quark masses using the SVZ QCD spectral sum rules (QSSR) [5, 6], lattice data [7] and LEP experiments¹. In this note, I propose some new τ -like decay sum rules for the extraction of the strange-quark mass from the ϕ -meson, and τ -decay data, which, unlike the sum rule proposed in [10] (hereafter referred as SN) involving the difference of the isoscalar and isovector component of the $e^+e^- \rightarrow$ hadrons data, are not directly affected by the less (theoretically) controlled effect of the ω - ρ mixing. We shall see later on that the result of the present analysis surprisingly agrees quite well with the previous result in SN assuming $SU(2)$ isospin symmetry (neglect of the ω - ρ mixing effect). This fact then signals small effects of $SU(2)$ violation due to the ω - ρ mixing parameters in the sum rules involving the difference of the isoscalar and isovector components of the $e^+e^- \rightarrow$ hadrons data. It also questions the reliability of the existing sum rule estimates of these parameters [11] following the pioneer work of SVZ [12], and the uses of such estimates for extracting m_s [13].

2 Normalizations and notation

We shall be concerned with the transverse two-point correlator:

$$\Pi_{ab}^{\mu\nu}(q) \equiv i \int d^4x e^{iqx} \langle 0 | \mathcal{T} J_a^\mu(x) (J_b^\nu(0))^\dagger | 0 \rangle = -(g_{\mu\nu}q^2 - q_\mu q_\nu) \Pi_{ab}(q^2) , \quad (1)$$

built from the $SU(3)$ component of the local electromagnetic current:

$$J_{EM}^\mu = V_3^\mu(x) + \frac{1}{\sqrt{3}} V_8^\mu(x) , \quad (2)$$

where:

$$V_a^\mu(x) \equiv \sqrt{\frac{1}{2}} \bar{\psi}(x) \lambda_a \gamma^\mu \psi(x) ; \quad (3)$$

λ_a are the diagonal flavour $SU(3)$ matrices:

$$\lambda_3 = \sqrt{\frac{1}{2}} \begin{pmatrix} 1 & & \\ & -1 & \\ & & 0 \end{pmatrix} , \quad \lambda_8 = \sqrt{\frac{1}{6}} \begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix} , \quad (4)$$

acting on the basis defined by the up, down and strange quarks:

$$\psi(x) = \begin{pmatrix} u(x) \\ d(x) \\ s(x) \end{pmatrix} . \quad (5)$$

In terms of the diagonal quark correlator:

$$\Pi_{jj}^{\mu\nu}(q) \equiv i \int d^4x e^{iqx} \langle 0 | \mathcal{T} J_j^\mu(x) (J_j^\nu(0))^\dagger | 0 \rangle = -(g_{\mu\nu}q^2 - q_\mu q_\nu) \Pi_{jj}(q^2) \quad j = u, d, s , \quad (6)$$

the previous $SU(3)$ flavour components of the electromagnetic correlator read²:

$$\begin{aligned} \Pi_{33} &= \frac{1}{2} \cdot \frac{1}{2} (\Pi_{uu} + \Pi_{dd}) , \\ \Pi_{88} &= \frac{1}{2} \cdot \frac{1}{6} (\Pi_{uu} + \Pi_{dd} + 2\Pi_{ss}) , \\ \Pi_{38} &= \frac{1}{4\sqrt{3}} (\Pi_{uu} - \Pi_{dd}) , \end{aligned} \quad (7)$$

¹For recent comparisons of different determinations, see e.g. [8] and [9].

²We shall follow the normalization used in SN [10].

Therefore, the $e^+e^- \rightarrow \text{hadrons}$ total cross-section reads:

$$\sigma(e^+e^- \rightarrow \text{hadrons})_{u,d,s} = \frac{4\pi^2\alpha}{s} e^2 \frac{1}{\pi} \left\{ \text{Im}\Pi_{33}(s) + \frac{1}{3}\text{Im}\Pi_{88}(s) + \frac{2}{\sqrt{3}}\text{Im}\Pi_{38}(s) \right\}. \quad (8)$$

In a narrow-width approximation (NWA), the resonance H contributions to the spectral functions can be introduced through:

$$\langle 0|V_a^\mu|H\rangle = \epsilon^\mu \frac{M_H^2}{2\gamma_{Ha}}, \quad (9)$$

where the coupling γ_{Ha} is related to the meson leptonic width as:

$$\Gamma_{H \rightarrow e^+e^-} = \frac{2}{3}\alpha^2 \pi \frac{M_H}{2\gamma_{Ha}^2}, \quad (10)$$

which is itself related to the total cross-section:

$$\sigma(e^+e^- \rightarrow H) = 12\pi^2 \frac{\Gamma_{H \rightarrow e^+e^-}}{M_H} \delta(s - M_H^2). \quad (11)$$

3 ϕ -meson τ -like decay sum rules

We shall be concerned with the hypothetical τ -like decay sum rules :

$$R_{\tau,\phi} \equiv \frac{3|V_{ud}|^2}{2\pi\alpha^2} S_{EW} \int_0^{M_\tau^2} ds \left(1 - \frac{s}{M_\tau^2}\right)^2 \left(1 + \frac{2s}{M_\tau^2}\right) \frac{s}{M_\tau^2} \sigma_{e^+e^- \rightarrow \phi, \phi', \dots}, \quad (12)$$

and the $SU(3)$ -breaking combinations:

$$\Delta_{1\phi} \equiv R_{\tau,1} - R_{\tau,\phi}, \quad (13)$$

and [10]:

$$\Delta_{10} \equiv R_{\tau,1} - 3R_{\tau,0}, \quad (14)$$

which involves the difference of the isoscalar ($R_{\tau,0}$) and isovector ($R_{\tau,1}$) sum rules à la Das-mathur-Okubo [14], and vanishes in the $SU(3)$ symmetry limit. Here, [15] $S_{EW} = 1.0194$ is the electroweak correction, $|V_{ud}|^2 = 0.975$ is the CKM mixing angle ³.

Its QCD expression is known to be [16] (hereafter referred as BNP):

$$R_{\tau,\phi} = \frac{3}{2}|V_{ud}|^2 S_{EW} \frac{2}{9} \left(1 + \delta_{EW} + \delta^{(0)} + \sum_{D=2,4,6} \frac{\delta_{ss}^{(D)}}{M_\tau^D}\right), \quad (15)$$

where [17]:

$$\delta_{EW} = 0.0010. \quad (16)$$

Using fixed-order perturbation theory (FOPT), one obtains partly from BNP:

$$\begin{aligned} \delta^{(0)} &= \left(\frac{\alpha_s}{\pi} \equiv a_s\right) + 5.2023a_s^2 + 26.366a_s^3 \pm 134a_s^4, \\ \delta_{ss}^{(2)} &= -2.1a_s\lambda^2 - 12\overline{m}_s^2 \left(1 + \frac{13}{3}a_s + 30.4a_s^2 + (d_3 \equiv k_3 - 45.147)a_s^3 \pm d_4a_s^4 - 2\frac{a_s\lambda^2}{M_\tau^2}\right), \\ \delta_{ss}^{(4)} &= a_s^2 \left[\frac{11}{4}\pi\langle\alpha_s G^2\rangle - 36\pi^2\langle m_s \bar{s}s\rangle - 8\pi^2 \sum_{q=u,d,s} \langle m_q \bar{q}q\rangle \right] + 36\overline{m}_s^4, \\ \delta_{ss}^{(6)} &\simeq 7\frac{256}{27}\rho\alpha_s\langle\bar{s}s\rangle^2. \end{aligned} \quad (17)$$

The expressions of the m_s^2 terms have been derived to order a_s^2 in SN and [13]. The order a_s^3 terms have been obtained by [18], using the results in [19]. The other higher dimension terms come from SN using the BNP's result. We have added, into the expression, the contribution of the tachyonic gluon

³One should notice that this term is an overall factor appearing in both sides of the sum rules. It has been only introduced in order to mimic the expression of the true τ -decay width in the charged current sector.

mass λ^2 motivated to mimic the unknown large order terms (renormalons,...) of the QCD perturbative asymptotic series [22], where [23, 24]:

$$a_s \lambda^2 \simeq -(0.06 \sim 0.07) \text{GeV}^2 . \quad (18)$$

In order to be conservative, we shall consider this effect as a source of systematic errors. One can estimate [18] the constant term $k_3 a_s^3$ of the two-point correlator, by assuming a “naïve” geometric growth of the perturbative coefficients, as usually done (BNP, [20, 21]). Assuming a 50% error in this estimate, one then obtains: $k_3 \approx 24.1415^2/2.6667 \simeq (218.554 \pm 109.268)$. This leads to the value:

$$d_3 = 173.4 \pm 109.2 , \quad (19)$$

of the full coefficient of the a_s^3 -term. Alternatively, a much more naïve estimate of the a_s^3 coefficient d_3 can be obtained from an assumed geometric growth of the perturbative coefficients of the $R_{\tau,\phi}$ perturbative series, after the uses of the contour integral. In this way, one obtains, a value $d_3 \simeq 30.410^2/4.333 \simeq 208$, which is consistent with the previous number. Therefore, we consider the error estimate given previously, as a generous estimate. We add to this error estimate, the one due to the unknown a_s^4 , which we have again estimated from a naïve geometric growth of the perturbative coefficients (134 for $\delta^{(0)}$ and $d_4 = d_3^2/30 \approx 1002$ for $\delta_{ss}^{(2)}$).

The expression of the running coupling to three-loop accuracy can be parametrized as [6]:

$$\begin{aligned} a_s(\nu) &= a_s^{(0)} \left\{ 1 - a_s^{(0)} \frac{\beta_2}{\beta_1} \log \log \frac{\nu^2}{\Lambda^2} \right. \\ &+ \left. \left(a_s^{(0)} \right)^2 \left[\frac{\beta_2^2}{\beta_1^2} \log^2 \log \frac{\nu^2}{\Lambda^2} - \frac{\beta_2^2}{\beta_1^2} \log \log \frac{\nu^2}{\Lambda^2} - \frac{\beta_2^2}{\beta_1^2} + \frac{\beta_3}{\beta_1} \right] + \mathcal{O}(a_s^3) \right\} , \end{aligned} \quad (20)$$

with:

$$a_s^{(0)} \equiv \frac{1}{-\beta_1 \log(\nu/\Lambda)} , \quad (21)$$

and β_i are the $\mathcal{O}(a_s^i)$ coefficients of the β function in the \overline{MS} -scheme for n_f flavours, which read, for three flavours:

$$\beta_1 = -9/2 , \quad \beta_2 = -8 , \quad \beta_3 = -20.1198 . \quad (22)$$

Λ is a renormalization-group-invariant (RGI) scale, but is renormalization-scheme-dependent. The expression of the running quark mass in terms of the invariant mass \hat{m}_i is [6]:

$$\begin{aligned} \overline{m}_i(\nu) &= \hat{m}_i (-\beta_1 a_s(\nu))^{-\gamma_1/\beta_1} \left\{ 1 + \frac{\beta_2}{\beta_1} \left(\frac{\gamma_1}{\beta_1} - \frac{\gamma_2}{\beta_2} \right) a_s(\nu) \right. \\ &+ \frac{1}{2} \left[\frac{\beta_2^2}{\beta_1^2} \left(\frac{\gamma_1}{\beta_1} - \frac{\gamma_2}{\beta_2} \right)^2 - \frac{\beta_2^2}{\beta_1^2} \left(\frac{\gamma_1}{\beta_1} - \frac{\gamma_2}{\beta_2} \right) + \frac{\beta_3}{\beta_1} \left(\frac{\gamma_1}{\beta_1} - \frac{\gamma_3}{\beta_3} \right) \right] a_s^2(\nu) \\ &+ \left. 1.95168 a_s^3 \right\} , \end{aligned} \quad (23)$$

where the a_s^3 term comes from [25]; γ_i are the $\mathcal{O}(a_s^i)$ coefficients of the quark-mass anomalous dimension, which read for three flavours:

$$\gamma_1 = 2 , \quad \gamma_2 = 91/12 , \quad \gamma_3 = 24.8404 . \quad (24)$$

In the present analysis, we shall use as inputs [4], [26]–[28]:

$$\Lambda = (375 \pm 50) \text{ MeV} , \quad (25)$$

and [29, 6]:

$$\begin{aligned} \langle \alpha_s G^2 \rangle &\simeq (0.07 \pm 0.01) \text{ GeV}^4 , \\ m_{u,d} \langle \bar{u}u \rangle &\simeq -\frac{1}{2} m_\pi^2 f_\pi^2 , \\ \langle \bar{s}s \rangle / \langle \bar{u}u \rangle &\simeq (0.7 \pm 0.1) , \\ \rho \alpha_s \langle \bar{u}u \rangle^2 &\simeq (5.8 \pm 0.9) \times 10^{-4} \text{ GeV}^6 . \end{aligned} \quad (26)$$

For the phenomenological estimate of the $R_{\tau,\phi}$ sum rule, we introduce, as stated before, the contributions of the resonances using a NWA, where we use [4]:

$$\Gamma_{\phi(1019) \rightarrow e^+ e^-} = (1.32 \pm 0.04) \text{ keV} , \quad \Gamma_{\phi(1680) \rightarrow e^+ e^-} = (0.48 \pm 0.14) \text{ keV} . \quad (27)$$

Table 1: Phenomenological estimates of $R_{\tau,I}$, $\Delta_{1\phi}$ and central values of \overline{m}_s to order α_s^3

M_τ [GeV]	$\frac{9}{2}R_{\tau,\phi}$	$\overline{m}_s(1 \text{ GeV})$ [MeV]	$R_{\tau,1}$ from [27]	$\Delta_{1\phi}$	$\overline{m}_s(1 \text{ GeV})$ [MeV]
1.2	0.85 ± 0.04	223.3	2.03 ± 0.025	1.18 ± 0.05	222.5
1.4	1.47 ± 0.06	186.1	1.95 ± 0.025	0.48 ± 0.07	193.1
1.6	1.59 ± 0.05	173.1	1.85 ± 0.025	0.26 ± 0.06	185.8
1.8	1.54 ± 0.07	207.6	1.78 ± 0.025	0.24 ± 0.07	226.5

Table 2: Different sources of errors in units of [MeV] for the estimate of \overline{m}_s (1 GeV)

Sources	$ \Delta m_s $ from $R_{\tau,\phi}$	$ \Delta m_s $ from $\Delta_{1\phi}$
<u>Experiment</u>		
Data	19	26
Mixing angle $(35 \pm 1)^0$	0.8	0.8
Total exp. (quadratic)	19	25
<u>Theory</u>		
$\Lambda = (375 \pm 50) \text{ MeV}$	15	8
$\pm 134\alpha_s^4$	11	—
$\pm 12m_s^2 \times 109.3\alpha_s^3$	6	6
$\pm 12m_s^2 \times 1002\alpha_s^4$	6	8
FOPT or CIE series	8	13
tachyonic gluon mass	15	2
$\langle \alpha_s G^2 \rangle = (0.07 \pm 0.01) \text{ GeV}^4$	0.04	0
$\langle \bar{s}s \rangle / \langle \bar{u}u \rangle = 0.7 \pm 0.1$	6	4
$d = 6$ condensates	3	2
Total theory (quadratic)	27	19

The continuum contribution (which is relatively small) has been estimated using a SU(3) symmetry relation on the results obtained in SN for the isoscalar sum rule. The sum of the different contributions is given in Table 1.

In our estimate, we include the $m_s^2\alpha_s^3$ term correction with the coefficient estimated previously, which is in the range of the value obtained for the charged current sector [31, 30]. However, we expect that the estimate of the error done in this way can be an overestimate of the real theoretical error.

Another eventual source of error is the small deviation from the ideal mixing angle $\theta_V = 35.3^0$, which

experimentally is [4]: $\theta_V \simeq 35^\circ$ ⁴. We take into account this deviation by assuming that it introduces an error of about $0.01 \times R_{\tau,\phi}$ on the value of $R_{\tau,\phi}$ given in Table 1. This effect is much smaller than the experimental error in the estimate of $R_{\tau,\phi}$ and $\Delta_{1\phi}$ given in Table 1, and leads to a small error (see Table 2).

4 \overline{m}_s from $R_{\tau,\phi}$: estimate and upper bound

Using the previous inputs, we give to order α_s^3 , the central value of $\overline{m}_s(1 \text{ GeV})$ from $R_{\tau,\phi}$ in the 3rd column of Table 1. We consider, as an optimal result, the one obtained for $M_\tau = 1.6 \text{ GeV}$, where the estimate presents a minimum, which is an indication of a compromise region where both the continuum + higher-states contributions and the non-perturbative + higher-order terms of the QCD series to the sum rules are (reasonably) small. The different sources of errors for this value are given in Table 2. One may check the convergence of the QCD perturbative series at this scale $M_\tau=1.6 \text{ GeV}$. Using the corresponding value of $a_s(1.6 \text{ GeV}) \simeq 0.114$, one can deduce each terms of the perturbative series :

$$\begin{aligned} 1 + \delta^{(0)} &\simeq 1 + 0.114 + 0.068 + 0.04 \pm 0.02 + \dots \\ \delta_{ss}^{(2)}|_{m_s} &\simeq -12\overline{m}_s^2 \left[1 + 0.494 + 0.390 + (0.283 \pm 0.178) \pm 0.169 + \dots \right], \end{aligned} \quad (28)$$

which shows a slow (but not a catastrophic) convergence. The last error is due to the estimate based on the geometrical growth of the coefficients of the unknown a_s^4 term in the series. The error ± 0.178 is the assumed uncertainty of the d_3 coefficient. However, the convergence of the QCD series is much better here than in the case of the charged currents [30, 31]. In the present analysis, we consider that the remaining terms of the series can be mimiced by the tachyonic gluon mass introduced previously in [22, 23], where a detailed explanation of the physical role of this non-standard term is discussed. In order, to avoid a misinterpretation of our result, we include again this effect into the theoretical error, where a such tachyonic mass contribution is mainly important in the unflavoured term and tends to increase the mass of the strange quark by about 9%, while it is almost negligible of about 1% in the flavoured $SU(3)$ breaking terms [23]. Within the previous four different sources (including the one due to the treatment of the QCD series: fixed order (FOPT) [16] or contour improved expansion (CIE) expected to improve the convergence of the QCD series [33]) of theoretical uncertainties which we have included (see Table 2), we expect that the theoretical errors given in Table 2 are largely overestimated. Taking into account our previous discussions and the fact that the signs of each a_s corrections are all positive, the truncated PT series is expected to be a quite good approximation of the full series. One should also remember that, for the quantity of interest m_s , the corrections are about half of the one appearing in the PT series above. From the previous analysis, and after solving a quartic equation in m_s , the $R_{\tau,\phi}$ sum rule leads to the result (to order α_s^3) given in Table 1 in the entry corresponding to $M_\tau = 1.6 \text{ GeV}$, where we only give the central value of the strange quark mass coming from a mean value of the one from FOPT and CIE (here, the result from CIE is slightly higher than from FOPT by 10 MeV). Collecting all different sources of errors in Table 2, we consider, as a final estimate:

$$\overline{m}_s(1 \text{ GeV}) \simeq (173 \pm 19_{exp} \pm 27_{th}) \text{ MeV} . \quad (29)$$

We can also use the positivity of the spectral function and retains only the lowest ground state ϕ -meson contribution in order to derive an upper bound for m_s . The absolute (weakest) bound comes from the inclusion of the tachyonic gluon contribution into the QCD expression and using the CIE. It reads:

$$\overline{m}_s(1 \text{ GeV}) \leq (200 \pm 17_{exp} \pm 22_{th}) \text{ MeV} , \implies \overline{m}_s(2 \text{ GeV}) \leq (145 \pm 12_{exp} \pm 17_{th}) \text{ MeV} , \quad (30)$$

which is, however, stronger than the one obtained in [34] from the pseudoscalar moment sum rules.

5 \overline{m}_s from $\Delta_{1,\phi}$

We use the more recent and precise value of $R_{\tau,1}$ from Fig. 17 of ALEPH [27] (see also OPAL [28]), which agrees within the errors with the one estimated in SN and in [35, 21] from $e^+e^- \rightarrow \text{hadrons}$ data. Then, we consider the $SU(3)$ -breaking τ -like sum rule:

$$\Delta_{1\phi} \equiv R_{\tau,1} - R_{\tau,\phi}, \quad (31)$$

⁴Some theoretical estimates from the off-diagonal mass matrix leads to slightly higher values of the mixing angle [4].

which, as the one in SN:

$$\Delta_{10} \equiv R_{\tau,1} - 3R_{\tau,0} , \quad (32)$$

which involves the difference of the isoscalar and isovector spectral function à la Das-mathur-Okubo [14], vanishes in the $SU(3)$ symmetry limit. Its advantage over the previous $R_{\tau,\phi}$ sum rule is the vanishing of the λ^2 tachyonic gluon-mass contribution to leading order, and its weaker sensitivity to the value of α_s . Here, the tachyonic gluon mass tends to decrease the strange quark-mass value by 1% [23]. The advantage of this sum rule compared with the difference of sum rules proposed in SN, is its weaker sensitivity to the strength of the $SU(2)$ isospin violation due to the ω - ρ mixing contribution. The phenomenological value of $\Delta_{1\phi}$ is given in the 5th column of Table 1, while the central value of \overline{m}_s (1 GeV) resulting from an average between the FOPT and CIE results (here, the CIE result is lower by about 26 MeV than the FOPT one) is given in the last column of this table. As in the case of the $R_{\tau,\phi}$ sum rule, the optimal result is obtained for $M_\tau \simeq 1.6$ GeV. The different sources of errors in the estimate is given in Table 2. The final result is:

$$\overline{m}_s(1 \text{ GeV}) \simeq (186 \pm 25_{exp} \pm 19_{th}) \text{ MeV} . \quad (33)$$

6 \overline{m}_s from $\Delta_{1,0}$: updated

The two results from the two sum rules are in a good agreement with each others. This result also agrees with the previous estimate in SN from the difference Δ_{10} between the isovector and isoscalar τ -like decay sum rule in e^+e^- , to order a_s^2 [10]:

$$\overline{m}_s(1 \text{ GeV}) \simeq (194 \pm 25_{exp} \pm 19_{th}) \text{ MeV} . \quad (34)$$

obtained by assuming a good realization of the $SU(2)$ isospin symmetry, i.e. by neglecting the ω - ρ mixing violating term. We update the previous estimate of SN by including the a_s^3 contribution and by using the recent value of $R_{\tau,1}$ from ALEPH [27] into Table 2 of SN. The strengths of the theoretical errors are about the same as the ones for $\Delta_{1,\phi}$ given in Table 2. In this way, one obtains, to order a_s^3 :

$$\overline{m}_s(1 \text{ GeV}) \simeq (176 \pm 24_{exp} \pm 19_{th}) \text{ MeV} . \quad (35)$$

The agreement of this result with the two former ones in Eqs (29) and (33) rules out definitely unusual huge $SU(2)$ violation induced by the ω - ρ mixing term advocated in [13], which can decrease the previous value of m_s by about a factor 2. We shall comment on this point in the next sections.

7 Final value and comparison with other results

We take as our final result the mean value of the three different results given in Eqs. (29), (33) and (35), and take as a conservative error, the largest one from the three determinations, which is in Eq. (29). In this way, we obtain:

$$\overline{m}_s(1 \text{ GeV}) \simeq (178 \pm 33) \text{ MeV} , \implies \overline{m}_s(2 \text{ GeV}) \simeq (129 \pm 24) \text{ MeV} . \quad (36)$$

This result agrees within the errors with the one from the published results for the $\Delta S = -1$ component of τ -decay [27, 31]⁵:

$$\overline{m}_s(1 \text{ GeV}) \simeq (234_{-64_{exp}-37_{theo}}^{+49_{exp}+32_{th}} \pm 10.6_{fit} \pm 14.6_{J=0}) \text{ MeV} . \quad (37)$$

However, our result is much more accurate due to the much better convergence of the QCD series in the neutral vector current channel, but also to the more precise measurement of the sum rule in the neutral channel where the contribution is dominated by the quite accurate measurement of the ϕ leptonic width, and of the new accurate value of $R_{\tau,1}$ from recent ALEPH data. Further improvements of our result need much more accurate measurements of the e^+e^- $I=0$ data or/and the leptonic widths of the different ϕ families, but also improved control of the QCD perturbative series. The agreement of the result with the one from τ -decay also indicates a good realization of the $SU(2)$ symmetry relating the charged and neutral currents (CVC). This result also agrees with some of the results from the (pseudo)scalar sum rules [6, 9, 34], [36]–[39] and from a global fit [40] of the light quark condensates from the hadron sector. The agreements of these different results raise the questions of the reliability of the available estimates [11] of the parameters of the ω - ρ mixing based on the SVZ sum rules [12], and rule out a violent eventual reduction of about a factor 2 [13] on the previous estimate in Eq (35).

⁵During the submission of this revised version of the paper, a recent analysis [35] obtaining a value similar to ours has been published.

8 On the ω - ρ mixing from the SVZ sum rules

This quantity is controlled by the $SU(2)$ -violating off-diagonal Π_{08} correlator defined in Eq. (7). In [12], SVZ have estimated this quantity using QCD spectral sum rules. The sum rule reads:

$$\int dt e^{-t\tau} \frac{1}{\pi} \Pi_{08}(t) \simeq \tau^{-1} \{ \delta_0 + \delta_2 \tau + \delta_4 \tau^2 + \delta_6 \tau^3 \} \quad (38)$$

where:

$$\begin{aligned} \delta_0 &= \frac{\alpha}{16\pi^3}, \\ \delta_2 &= \frac{3}{2\pi^2} (m_u^2 - m_d^2) \\ \delta_4 &= \frac{md - m_u}{m_d + m_u} 2m_\pi^2 f_\pi^2 \left(1 + \frac{md + m_u}{m_d - m_u} \frac{\gamma}{2 + \gamma} \right) \\ \delta_6 &= -\frac{224}{81} \pi \rho \alpha_s \langle \bar{u}u \rangle^2 \left(-\gamma + \frac{\alpha}{8\alpha_s} \right), \end{aligned} \quad (39)$$

where $\alpha = 1/137.036$ is the electromagnetic coupling and [6]:

$$\gamma \equiv \langle \bar{d}d \rangle / \langle \bar{u}u \rangle \simeq 1 - 9 \times 10^{-3}. \quad (40)$$

An analysis of the r.h.s of the sum rule shows that its τ -stability is reached at small values of $\tau \simeq 0.2 \text{ GeV}^{-2}$. This is a special feature compared with the case of the ρ -meson sum rule, which stabilizes at the typical meson mass scale $\tau \simeq M_\rho^{-2}$, where the lowest ground state contribution to the spectral integral dominates. This fact confirms SVZ previous arguments that the radial excitations $\omega'-\rho', \dots$ -mesons play an important role in this analysis, and they should affect the lowest ground state contributions in a significant way. The sensitivity of the analysis to the higher meson masses and to the finite width effect of the ρ -meson should render the estimate [11] (which comes from the cancellation of large numbers (see also [41])) quite inaccurate. These different features indicate that the estimate of the errors in the ω - ρ -mixing analysis [11] should have been underestimated or/and the analysis needs a much better comprehensive study. Effects of higher mass states should be more pronounced in the different finite energy sum rule (FESR) versions of the Laplace transform sum rules proposed so far [11], as well as in its τ -derivative (higher moments), which are needed in order to fix the complete set of the ω - ρ mixing parameters.

9 Summary and implications on the values of ϵ'/ϵ and of $m_{u,d}$

- We have estimated the strange-quark running mass using some new τ -like ϕ -meson sum rules, which are not affected by the eventual $SU(2)$ breaking due to the ω - ρ -mixing. Our final result is given in Eq. (36), which is:

$$\overline{m}_s(1 \text{ GeV}) \simeq (178 \pm 33) \text{ MeV}, \implies \overline{m}_s(2 \text{ GeV}) \simeq (129 \pm 24) \text{ MeV}, \quad (41)$$

while the positivity of the spectral function gives the upper bound in Eq. (30):

$$\overline{m}_s(1 \text{ GeV}) \leq (200 \pm 28) \text{ MeV}, \implies \overline{m}_s(2 \text{ GeV}) \leq (145 \pm 20) \text{ MeV}, \quad (42)$$

- The agreement of this result with the ones given particularly in SN1 and updated in section 6, and the different sum rules results from different channels [6, 10, 30, 31, 34], [36]–[39], rules out a huge effect due to the ω - ρ -mixing in the determination of m_s from the Δ_{10} sum rule, where this effect tends to reduce considerably the m_s value [13]. Therefore, this result questions the validity of the different estimates of the ω - ρ -mixing parameters based on the SVZ sum rule done in [11], which unlike other sum rules are very sensitive to the high-energy behaviour of the spectral function, as we have seen previously.

- Our result in Eq. (36) is in agreement within the errors, with the different results obtained from various sum rule channels [4, 6, 9] and lattice simulations in the quenched approximation [7, 9], though its central value is slightly higher than the sum rule plus chiral perturbation theory “grande average” [9] of $\overline{m}_s(2 \text{ GeV}) \simeq (119 \pm 16) \text{ MeV}$.

- Due to the difficulty for a precise determination of the non-perturbative four-quark matrix elements $Q_6 \approx B_6^{1/2}/m_s^2$ and $Q_8 \approx B_8^{3/2}/m_s^2$ controlling the ratio of the CP-violating quantity ϵ'/ϵ [3], which one expects to be more difficult than the extraction of m_s from the two-quark current correlator, we can combine the previous value of m_s with the data on $\epsilon'/\epsilon = (28 \pm 4) \times 10^{-4}$ [2], in order to get an estimate of the combination $Q_6 - 0.54Q_8$ within the standard model. Using the expression given in [3] and the value $\text{Im } \lambda_t \simeq (1.34 \pm 0.30) \times 10^{-4}$ from a standard analysis of the unitarity triangle, one can easily deduce for the value of m_s in Eq. (36):

$$B_6^{1/2} - 0.54B_8^{3/2} \simeq (2.8 \pm 1.3) , \quad (43)$$

which is a useful phenomenological extraction of these matrix elements, and which can serve as a guide in their non-perturbative determinations. This estimate may suggest a large violation of the vacuum saturation estimate of $B_6^{1/2} \simeq B_8^{3/2} \approx 1$ if negligible new physics affect the expression of ϵ'/ϵ . However, such a violation may not be very surprising because analogous violations of the vacuum saturation estimate of about a factor 2–3 has been already observed in the estimate of the four-quark condensates within the sum rule approach of the meson and baryon channels and in the analysis of the e^+e^- and τ -decay data [6, 29, 27, 28].

- Combining the result in Eq. (36) with the one:

$$(\overline{m}_u + \overline{m}_d)(1 \text{ GeV}) = (12 \pm 2) \text{ MeV} \quad \Longrightarrow \quad (\overline{m}_u + \overline{m}_d)(2 \text{ GeV}) = (8.7 \pm 1.5) \text{ MeV}, \quad (44)$$

from the most recent pseudoscalar sum rule analysis [37, 23], and from the direct extraction of the light quark condensate [40] ⁶, one obtains:

$$r_3 \equiv \frac{2m_s}{m_u + m_d} = 30 \pm 7 , \quad (45)$$

which is a non-trivial test of the chiral perturbation theory result of 24.4 ± 1.5 [8], that may be affected by some other ambiguities [42].

- Pursuing the update of the SN [10] result, one can introduce the previous ratio into the Dashen formula including NLO chiral corrections [43]:

$$r_2 \equiv \frac{m_d - m_u}{m_d + m_u} = (0.52 \pm 0.05)10^{-3}(r_3^2 - 1) \simeq 0.47 \pm 0.16 , \quad (46)$$

from which one obtains to order a_s^3 :

$$(\overline{m}_d - \overline{m}_u)(1 \text{ GeV}) = (5.6 \pm 2.1) \text{ MeV} \quad \Longrightarrow \quad (\overline{m}_d - \overline{m}_u)(2 \text{ GeV}) = (4.1 \pm 1.5) \text{ MeV} , \quad (47)$$

which one may compare with the lower bound $(\overline{m}_d - \overline{m}_u)(2 \text{ GeV}) \geq 1.5 \text{ MeV}$ obtained in [34]. Solving the previous equations, one can deduce:

$$\begin{aligned} \overline{m}_d(1 \text{ GeV}) &= (8.8 \pm 1.5) \text{ MeV} & \Longrightarrow & \quad \overline{m}_d(2 \text{ GeV}) = (6.4 \pm 1.1) \text{ MeV} , \\ \overline{m}_u(1 \text{ GeV}) &= (3.2 \pm 0.5) \text{ MeV} & \Longrightarrow & \quad \overline{m}_u(2 \text{ GeV}) = (2.3 \pm 0.4) \text{ MeV} , \end{aligned} \quad (48)$$

which we consider as an update of the previous sum rule results in [36, 10].

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⁶More complete discussions can be found in [9].

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